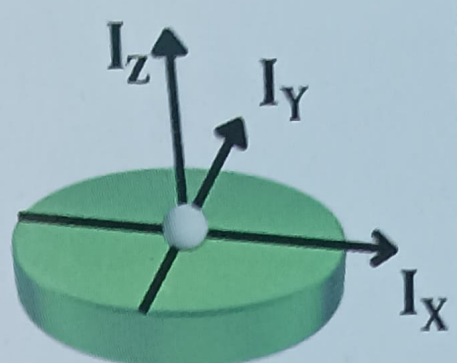
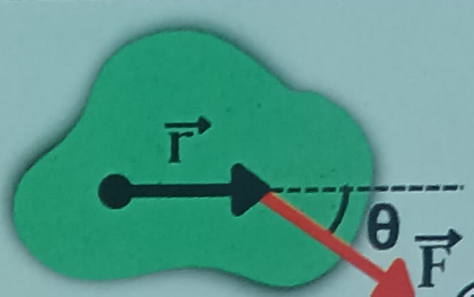
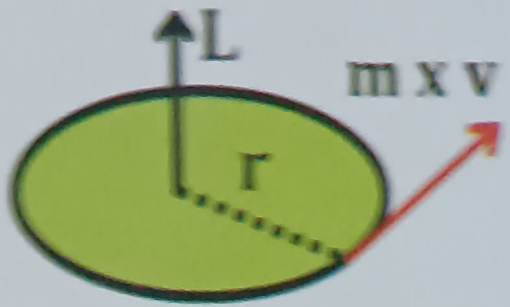
Rotational Motion

Centre of Mass of System of n Particles	$r_{CM} = \frac{\sum_{i=1}^{n} m_i r_i}{\sum m_i}$
Moment of Inertia	$I = \sum_{i=1}^{n} m_i r_i^2$
Parallel Axes Theorem  	$I = I_{CM} + Mr^2$ *I =Moment on inertia about axis* *I_{CM} = Moment of inertia about center of mass* *M = Mass of the body* *r=distance between two axis*
Perpendicular Axes Theorem  	$I_Z = I_X + I_Y$ *I_X, I_Y, I_Z are the moment of inertia along X, Y and Z axis*
Torque $\tau = r \times F = rF \sin\theta$	

Angular Momentum

$$L = I\omega = r \times mv$$



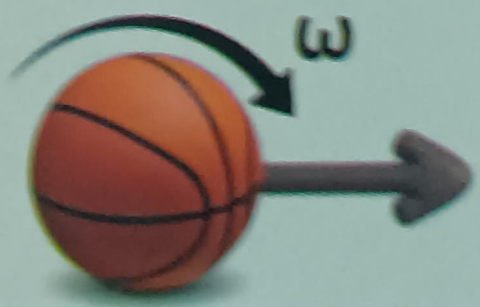
Conservation of Angular Momentum

If the external torque acting on a sysytem is zero, then its angular momentum remains constant.

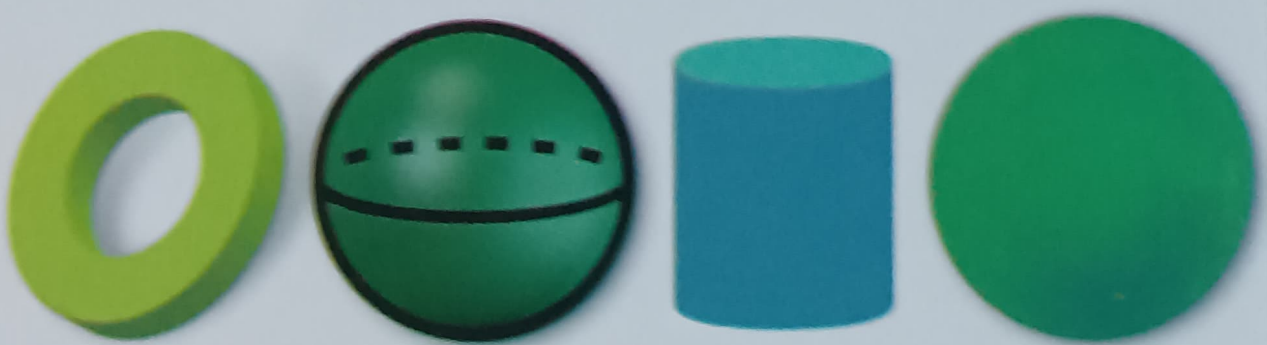
$$I_1\omega_1 = I_2\omega_2$$

Total kinetic energy of a rolling object

$$Energy = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$



Value of γ for different Objects



Ring	Hollow Sphere	Disc/Solid Cylinder	Solid Sphere
$I = MR^2$	$I = \frac{2}{3}MR^2$	$I = \frac{1}{2}MR^2$	$I = \frac{2}{5}MR^2$
$\gamma = \frac{1}{2} = 0.5$	$\gamma = \frac{3}{5} = 0.6$	$\gamma = \frac{2}{3} = 0.66$	$\gamma = \frac{5}{7} = 0.71$

Physical Quantity	Ring hollow cylinder	Hollow Sphere	Disc/Solid Cylinder	Solid Sphere
$\frac{K.E._{Translation}}{K.E._{Total}}$ γ	0.5	0.6	0.66	0.71
$\frac{K.E._{Rotation}}{K.E._{Total}}$ $1-\gamma$	0.5	0.4	0.33	0.29
$\frac{K.E._{Translation}}{K.E._{Rotation}}$ $\frac{\gamma}{1-\gamma}$	1:1	3:2	2:1	5:2
Acceleration on inclined surface $(a = g\sin\theta\,\gamma)$	$\frac{1}{2}g\sin\theta$	$\frac{3}{5}g\sin\theta$	$\frac{2}{3}g\sin\theta$	$\frac{5}{7}g\sin\theta$
Time taken to come down $t = \sqrt{\frac{2s}{g\sin\theta\gamma}}$	$\sqrt{\frac{4s}{g\sin\theta}}$	$\sqrt{\frac{10s}{3g\sin\theta}}$	$\sqrt{\frac{6s}{2g\sin\theta}}$	$\sqrt{\frac{14s}{5g\sin\theta}}$
Velocity at bottom of inclined surface $v = \sqrt{2gH\gamma}$	$\sqrt{gh}$	$\sqrt{\frac{6gH}{5}}$	$\sqrt{\frac{4gH}{3}}$	$\sqrt{\frac{10gH}{7}}$
H_{max} attained by particle $H = \frac{v^2}{2g\gamma}$	$H = \frac{v^2}{g}$	$H = \frac{5}{6}\frac{v^2}{g}$	$H = \frac{3}{4}\frac{v^2}{g}$	$H = \frac{7}{10}\frac{v^2}{g}$
Friction on inclined plane $= mg\sin\theta(1-\gamma)$	$\frac{1}{2}mg\sin\theta$	$\frac{2}{5}mg\sin\theta$	$\frac{1}{3}mg\sin\theta$	$\frac{2}{7}mg\sin\theta$
μ_{min} to start pure rolling $\mu = \tan\theta\,(1-\gamma)$	$\frac{1}{2}\tan\theta$	$\frac{2}{5}\tan\theta$	$\frac{1}{3}\tan\theta$	$\frac{2}{7}\tan\theta$

- Vehicle travels half the distance with speed v and the other dist. with speed 2v. Its Average speed is : **4v/3.**

- A bullet is fired from a gun at the speed of 280 ms-1 in the direction 30∘ above the horizontal. The maximum height attained by the bullet is :

- A horizontal bridge is built across a river. A student standing on the bridge throws a small ball vertically upwards with a velocity 4 ms^{-1}. The ball strikes the water surface after 4s. The height of bridge is : **64m**

- A football player is moving southward and suddenly turns eastward with the same speed to avoid an opponent. The force that acts on the player is : **Along North - East**

- Calculate the maximum acceleration of a moving car so that a body lying on the floor of the car remains stationary. The coefficient of static friction between the body and the floor is 0.15 : **1.5ms^{-2}**

- A bullet from a gun is fired on a rectangular wooden block with velocity u. When bullet travels 24 cm through the block along its length horizontally, velocity of bullet becomes u/3. Then it further penetrates into the block in the same direction before coming to rest at the other end of the block. The total length of the block is : **27 cm**